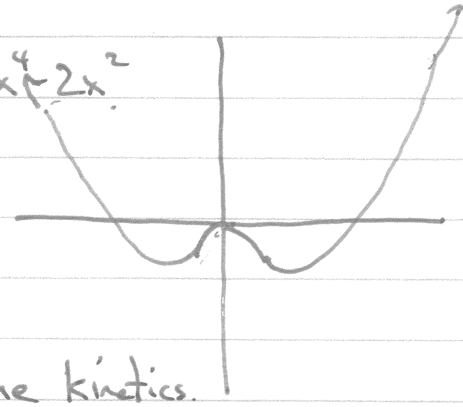
<u>Last time:</u> sketch simple polynomials

Ex. $f(x) = x^4 - 2x^2$



Near $x \approx 0$ lower deg dominates
As $x \to \infty$ higher deg "

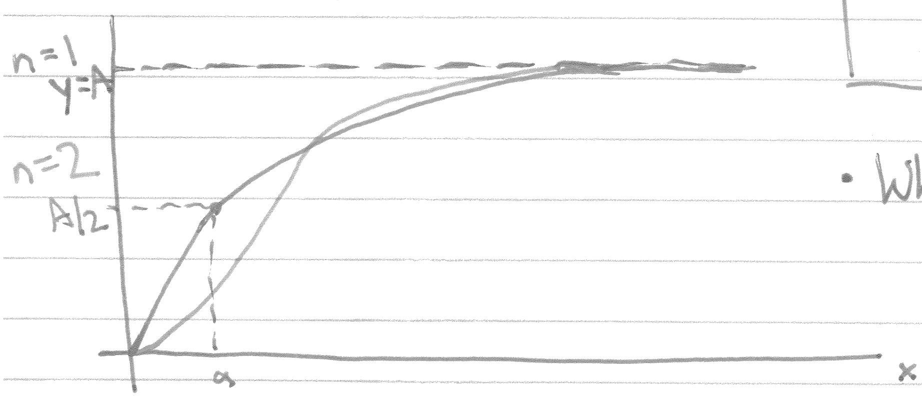
<u>Today:</u> · Apply to Hill functions, used to model enzyme kinetics.
· Averages rates of change of functions.

A <u>rational function</u> $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ & $q(x)$ are polynomials.

<u>Examples:</u> $\frac{2x+5}{3x^2-7}$, $\frac{x^3}{3+5x^2}$, ...

Hill function $f(x) = \frac{\boxed{A}x^n}{\boxed{a^n} + x^n}$

A, a constants



• When x is small,
$f(x) = \frac{Ax^n}{a^n + x^n} \approx \frac{A}{a^n} x^n$

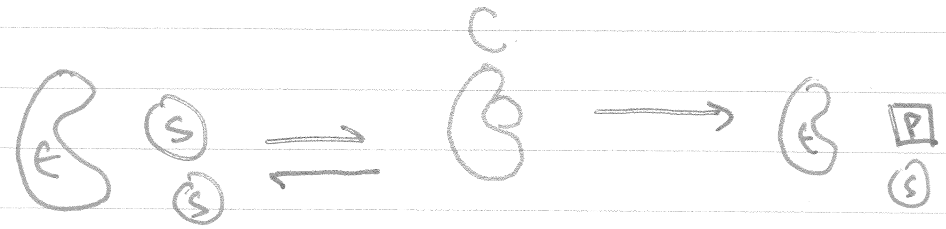
proportionally larger

• When x is large ($x >> a$)
$f(x) = \frac{Ax^n}{a^n + x^n} \approx \frac{Ax^n}{x^n} = A$

The line $y = A$ is an <u>asymptote</u>
$\lim_{x \to \infty} \frac{Ax^n}{a^n + x^n} = A$

$f(a) = \frac{Aa^n}{a^n + a^n} = \frac{A}{2}$ = half-maximal rate

Enzyme catalyzing a reaction (Michaelis–Menten kinetics)



Want an expression for
$f(x) =$ rate
$x =$ amt of substrate

- As you add more substrate, rate of reaction grows linearly
- As $x \to \infty$, you tend to a maximal rate.

Modeled by Hill function $f(x) = \dfrac{Ax^n}{a^n + x^n}$ — A: maximal rate, $n = 1$; a: amt of substrate giving half-maximal rate.

$E + S \rightleftharpoons C \longrightarrow E + P$

$E + 2S \rightleftharpoons C \longrightarrow E + 2P$ — $n = 2$

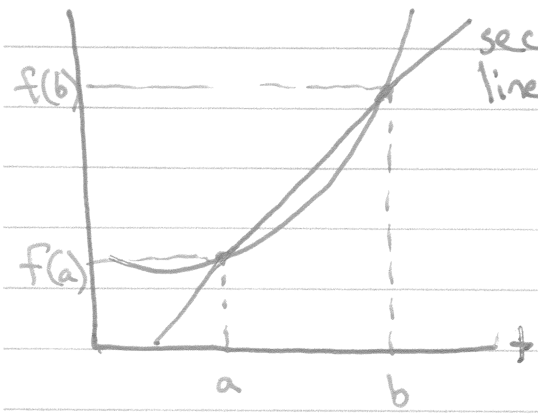
$n > 1$: "synergy" between substrates

Q: How to model
S_1 & S_2

$E + S_1 + S_2 \rightleftharpoons C \longrightarrow E + P_1 + P_2$

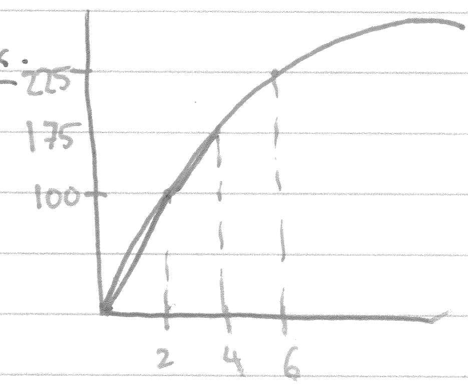
Average Rate of Change:

Let $f(t)$ be a function. The average rate of change over $a \le t \le b$ is the slope of secant line



$$\text{Slope} = \frac{\Delta f}{\Delta t} = \frac{f(b) - f(a)}{b - a}$$

Ex.



The average velocity over

$0 \le t \le 2$ is $\dfrac{100 - 0}{2 - 0} = 50$

$2 \le t \le 4$ is $\dfrac{175 - 100}{4 - 2} = \dfrac{75}{2} = 37.5$

Average velocity over an interval $\longleftrightarrow$ Secant lines

$\Downarrow$? limit as intervals get vanishingly small

Instantaneous velocity at a point $\longleftrightarrow$ tangent lines